AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

# GCSE MATHEMATICS

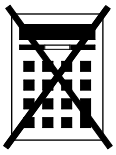
**H**

Higher Tier Paper 1 Non-Calculator

Tuesday 21 May 2019 Morning Time allowed: 1 hour 30 minutes

## Materials

For this paper you must have:
- mathematical instruments



You must **not** use a calculator.

## Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- Do all rough work in this book. Cross through any work you do not want to be marked.

## Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.

## Advice

In all calculations, show clearly how you work out your answer.

| For Examiner's Use | |
|---|---|
| Pages | Mark |
| 2–3 | |
| 4–5 | |
| 6–7 | |
| 8–9 | |
| 10–11 | |
| 12–13 | |
| 14–15 | |
| 16–17 | |
| 18–19 | |
| 20–21 | |
| 22–23 | |
| **TOTAL** | |

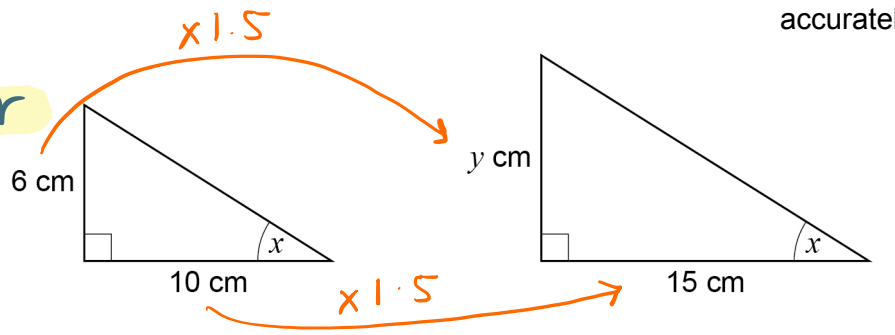
Answer **all** questions in the spaces provided

*Do not write outside the box*

**1** Here are two right-angled triangles.

Not drawn accurately

Find the scale factor




Circle the value of $y$.

**[1 mark]**

11 7.5 9 4

**2** Work out the value of $\left(1\frac{2}{3}\right)^2$

Change to improper.

$\left(\frac{5}{3}\right)^2 = \frac{25}{9}$

Circle your answer.

**[1 mark]**

$1\frac{4}{9}$ $3\frac{1}{3}$ $2\frac{4}{9}$ $2\frac{7}{9}$

**3** Work out the arc length, in metres, of a semicircle of radius 6 metres.

Recall Circumference = $\pi d$

Circle your answer.

**[1 mark]**

$3\pi$ $6\pi$ $12\pi$ $18\pi$

Circle → $\pi \times 12$

Semi-circle → $\pi \times 6$

*Do not write outside the box*

**4** Circle the fraction that is equivalent to 4.625

**[1 mark]**

$\frac{39}{8}$ $\frac{37}{8}$ $\frac{185}{4}$ $\frac{17}{4}$

Seperate $4.625 \rightarrow 4 + 0.625$

$0.625 = \frac{625}{1000} = \frac{5}{8}$ ∴ $4\frac{5}{8} \rightarrow \frac{37}{8}$

**5 (a)** Write 0.00097 in standard form.

**[1 mark]**

Answer $9.7 \times 10^{-4}$

**5 (b)** Work out $\frac{3 \times 10^5}{4 \times 10^3}$

Give your answer as an ordinary number.

**[2 marks]**

Cancel down

$0.75 \times 10^2$ (1)

$0.75 \times 100 = 75$

Change to ordinary number

$\frac{300000}{4000}$ (1) Cancel down

075

4|300

Answer 75 (1)

7

Do not write outside the box

**6** Anna plays a game with an ordinary, fair dice.

1/6 If she rolls 1 she wins.

2/6 If she rolls 2 or 3 she loses.

3/6 If she rolls 4, 5 or 6 she rolls again.

Find the probabilities

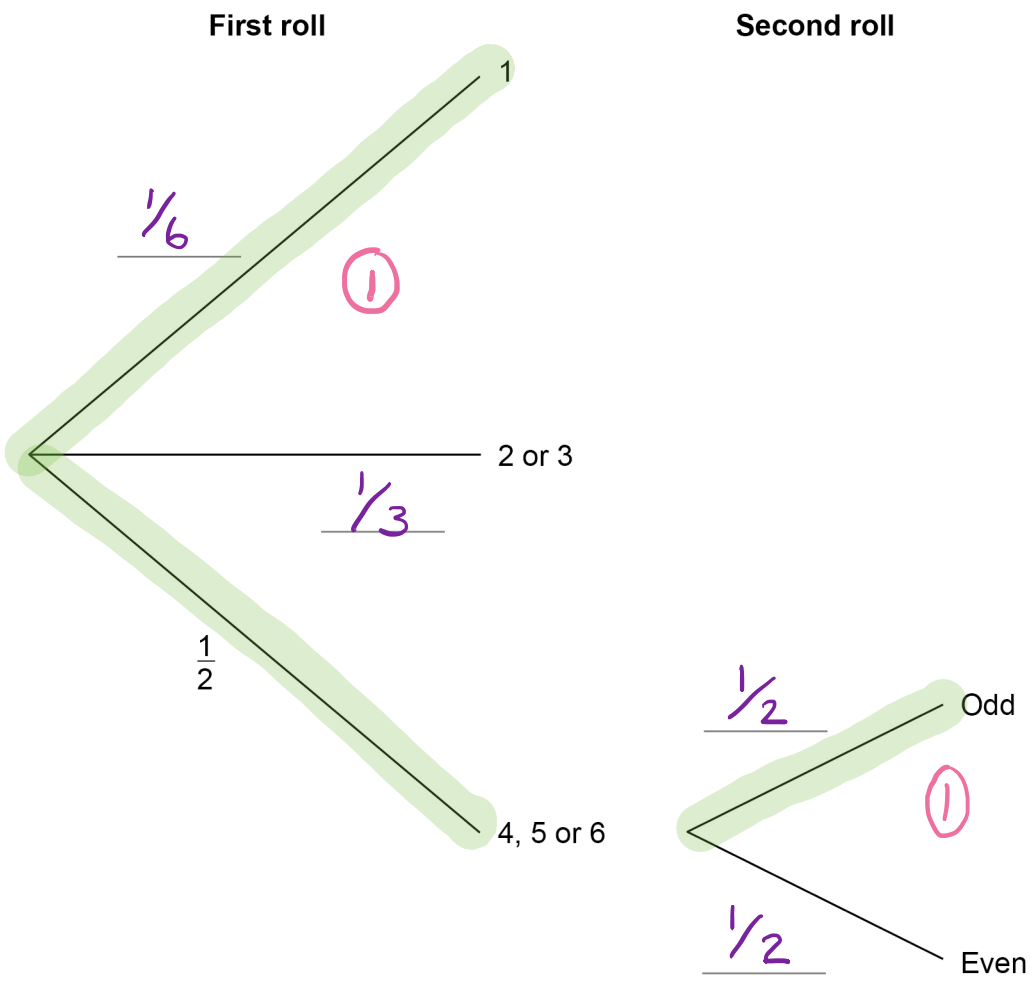
When she has to roll again,

1/2 if she rolls an odd number she wins

1/2 if she rolls an even number she loses.

**6 (a)** Complete the tree diagram with the four missing probabilities.

**[2 marks]**



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**6 (b)** Is Anna more likely to win or to lose?

You **must** work out the probability that she wins.

**[4 marks]**

Find the probability of winning

P(1) or P(4, 5 or 6 AND Odd)

(1)

$\frac{1}{6} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{6} + \frac{1}{4} = \frac{2}{12} + \frac{3}{12} = \frac{5}{12}$ (1)

Find the probability of losing

P(2 or 3) + P(4, 5, 6 AND Even)

(1)

$\frac{1}{3} + \frac{1}{2} \times \frac{1}{2} = \frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

∴ Anna is more likely to lose (1)

**Turn over for the next question**

6

*Do not write outside the box*

**7** Three friends arrive at a party.

Their arrival increases the number of people at the party by 20%

In total, how many people are now at the party?

**[2 marks]**

New arrivals → 20%

IF 3 = 20% 100% → 5 × 3 (1)

Answer 15 (1)

**8** Work out the value of $(3^{12} \div 3^5) \div (3^2 \times 3)$

**[3 marks]**

Break it down → find each of the brackets first

$3^{12} \div 3^5 = 3^7$ (1)

$3^2 \times 3 = 3^3$

$\therefore 3^7 \div 3^3 = 3^4$ (1)

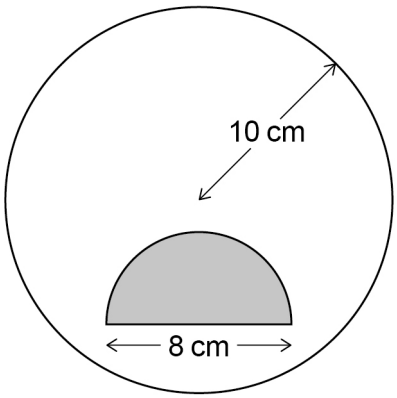
Work out $3^4$ for the final mark $3 \times 3 \times 3 \times 3$

Answer 81 (1)

Do not write outside the box

**9** A shaded semicircle is inside a circle as shown.

Not drawn accurately




The **radius** of the circle is 10 cm

The **diameter** of the semicircle is 8 cm

How many times bigger is the unshaded area than the shaded area?

**[4 marks]**

Find the area of the shaded and unshaded

NOTE: The unshaded is the circle minus the shaded semi-circle

Semi-circle $\rightarrow (\pi \times 4^2) \div 2 = 16\pi \div 2 = 8\pi$ (1)

Large circle $\rightarrow (\pi \times 10^2) = 100\pi$

Shaded $= 8\pi$ Unshaded $= 92\pi$ (1)

$\frac{92\pi}{8\pi}$ (1) $= \frac{92}{8} \rightarrow \frac{23}{2} = 11.5$

Answer 11.5 (1)

**Turn over for the next question**

9

Do not write outside the box

**10** The number of items, $n$, made in 1 hour by a machine is given by $n = \frac{60}{t}$

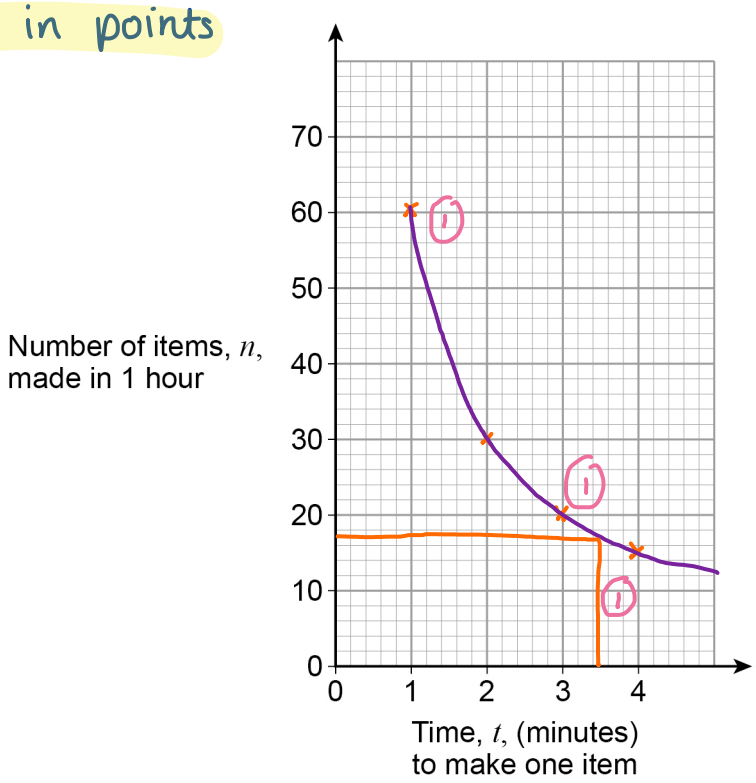
$t$ is the time in minutes the machine takes to make one item.

The value of $t$ changes for different types of item.

**10 (a)** On the grid below, draw the graph of $n = \frac{60}{t}$ for values of $t$ from 1 to 4

**[2 marks]**

Use the formula and sub in points




**10 (b)** The machine takes 3 minutes 30 seconds to make one item.

**Use your graph** to estimate the value of $n$.

**[2 marks]**

Answer 17 (1)

*Do not write outside the box*

**11** Ed and Fay shared £330 in the ratio 7 : 4

Ed gives Fay some of his money.

Fay now has the same amount as Ed.

How much does Ed give Fay?

**[3 marks]**

Share 330 in the ratio 7:4

330 ÷ 11 = 30 (1) 330 ÷ 2 = 165

Ed → 210 Fay → 120

Find the difference between half the amount and how much Fay will need

165 − 120 = 45

(1)

Answer £ 45 (1)

**12** The next term of a sequence is made by adding the previous two terms.

Which of these sequences follows this rule?

Circle your answer.

**[1 mark]**

Try each out!

–9 2 –7 –5 –12

–3 5 –2 3 1

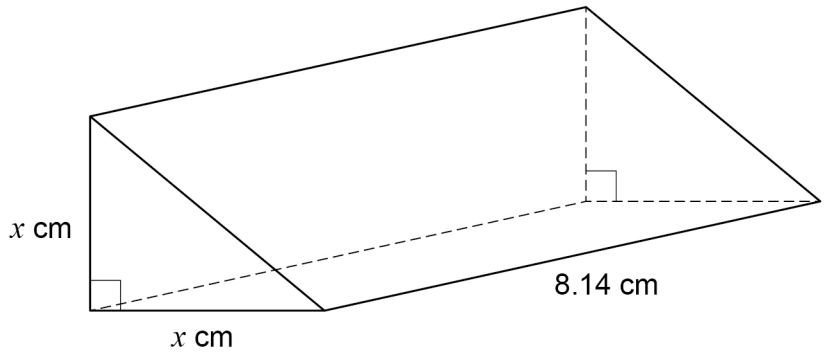
0 –3 –3 0 –3

–1 –1 –2 –3 1

8

Do not write outside the box

**13** The triangular cross section of a prism is an isosceles right-angled triangle.




The volume of the prism is 102 $cm^3$

Use approximations to estimate the value of $x$.

You **must** show your working.

**[3 marks]**

When approximating use 1 s.f.

Use Volume of a pyramid → $\frac{b \times h}{2} \times L$

(1)

$\frac{8x^2}{2} = 100 \rightarrow 4x^2 = 100$ (1)

$x^2 = 25$

$x = \sqrt{25}$

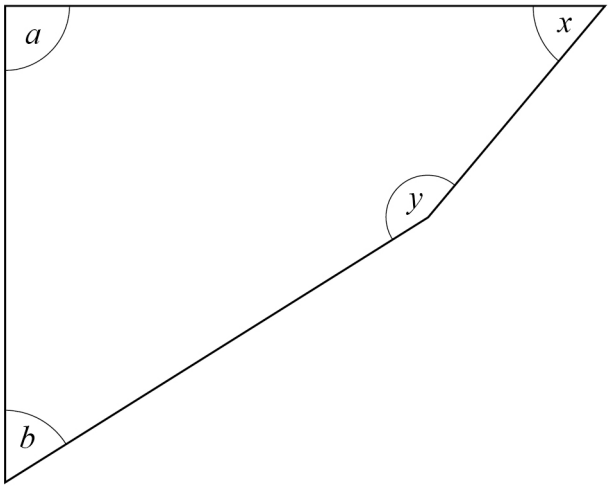
$x = 5$

discount -5 as its length

Answer 5

Do not write outside the box

**14** Here is a quadrilateral.




Not drawn accurately

$a = 90°$ and $a : b = 5 : 3$

$x : y = 1 : 3$

Show that $b = x$

**[3 marks]**

Do not use the fact $b = x$!

$a = 90$ $a : b \rightarrow 5 : 3$ (1)

so $90 \div 5$ (1) $= 18°$ $\therefore b = 54°$

Quadrilaterals add up to 180°

$360 - 90 - 54 = 216°$

$x : y \rightarrow 1 : 3$ so (1) $216 \div 4 = 54°$

$\therefore x = 54°$ $y = 162°$

$\therefore x = b = 54°$

6

**15** Here is some information about the test marks of 120 students.

| **Mark, *m*** | $0 < m \leqslant 10$ | $10 < m \leqslant 20$ | $20 < m \leqslant 30$ | $30 < m \leqslant 40$ | $40 < m \leqslant 50$ |
|---|---|---|---|---|---|
| **Frequency** | 20 | 28 | 40 | 20 | 12 |

Cumulative frequency = running total

**15 (a)** Complete the cumulative frequency table.

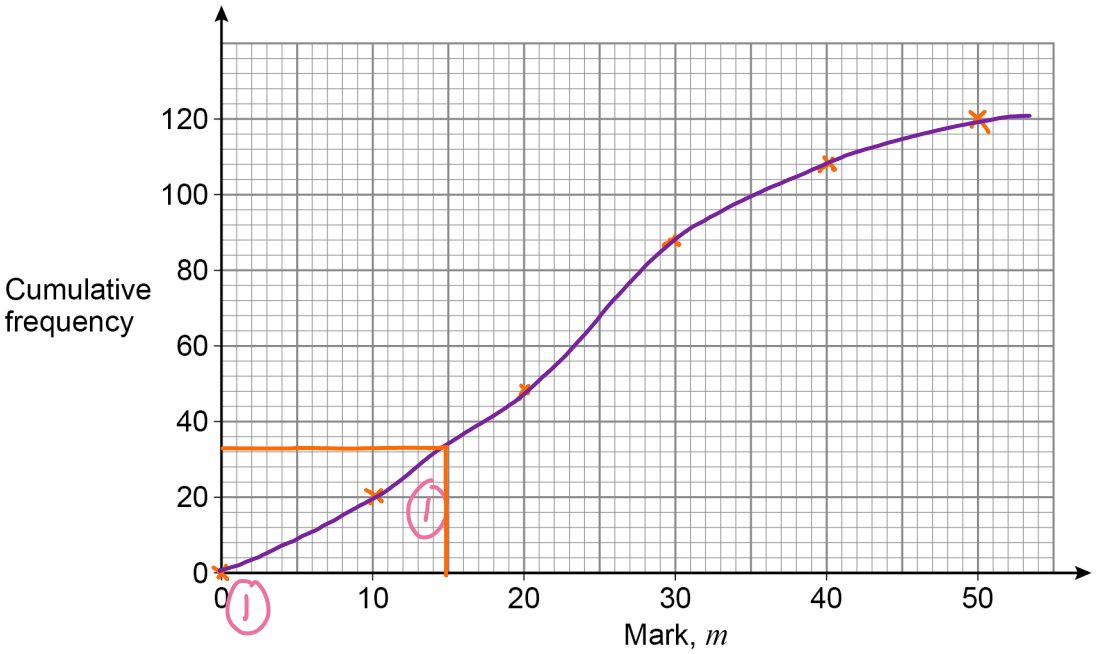
**[1 mark]**

| **Mark, *m*** | $m \leqslant 10$ | $m \leqslant 20$ | $m \leqslant 30$ | $m \leqslant 40$ | $m \leqslant 50$ |
|---|---|---|---|---|---|
| **Cumulative frequency** | 20 | 48 | 88 | 108 | 120 (1) |

plot at the upper value of each interval

**15 (b)** Draw a cumulative frequency graph.

**[2 marks]**



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**15 (c)** Students who scored 15 marks or fewer take another test.

Use your graph to estimate how many students take another test.

**[2 marks]**

Mark 15 on your graph (1)

Answer 33 (±2) (1)

**16** Simplify fully $\frac{4x - 8x^2}{12x - 6}$

**[3 marks]**

Algebraic fractions - often mean factorising

$\frac{4x(1-2x)}{6(2x-1)}$ (1) Almost the same brackets - we need to change the sign.

$\frac{-4x(2x-1)}{6(2x-1)}$ (1) $= \frac{-4x}{6} = \frac{-2x}{3}$

Answer $-2x/3$ (1)

**Turn over for the next question**

8

Do not write outside the box

**17** Toby is forming and solving equations.

**17 (a)**

The product of half of a number and three more than the number
is the same as
the square of the number

multiply

equals

Toby uses $y$ to represent the number.

Write an equation that Toby could form.

**[2 marks]**

$\frac{1}{2}y(y+3) = y^2 \longrightarrow \frac{1}{2}y^2 + \frac{3}{2}y = y^2$

$\frac{3}{2}y = \frac{1}{2}y^2$ (1)

Answer $1{\cdot}5y = 0{\cdot}5y^2$

**17 (b)** Toby forms another equation.

$$x = \frac{9}{8x}$$

He wants to work out the values of $x$.

Here is his working.

Take each step carefully

$x = \frac{9}{8x}$ $\times 8x$ ✓

$8x^2 = 9$ ✓

$8x = 3$ or $8x = -3$

$x = \frac{3}{8}$ or $x = -\frac{3}{8}$

What error has he made in his working?

**[1 mark]**

Toby has not square rooted the 8. (1)

Do not write outside the box

**18** Here is an identity.

$$x^2 - y^2 \equiv (x + y)(x - y)$$

**18 (a)** Use the identity to work out the value of $193^2 - 7^2$

You **must** show your working.

**[2 marks]**

$193^2 - 7^2 \equiv (193 + 7)(193 - 7)$

$\equiv 200 \times 86$ (1) Without the calculator, break it down.

$\equiv 17200$

Answer 17200 (1)

**18 (b)** Factorise $100a^2 - 81b^2$

**[1 mark]**

Difference between two squares

Answer $(10a - 9b)(10a + 9b)$ (1)

**19** Circle the fraction that is equivalent to $0.\dot{1}$

**[1 mark]**

(1)

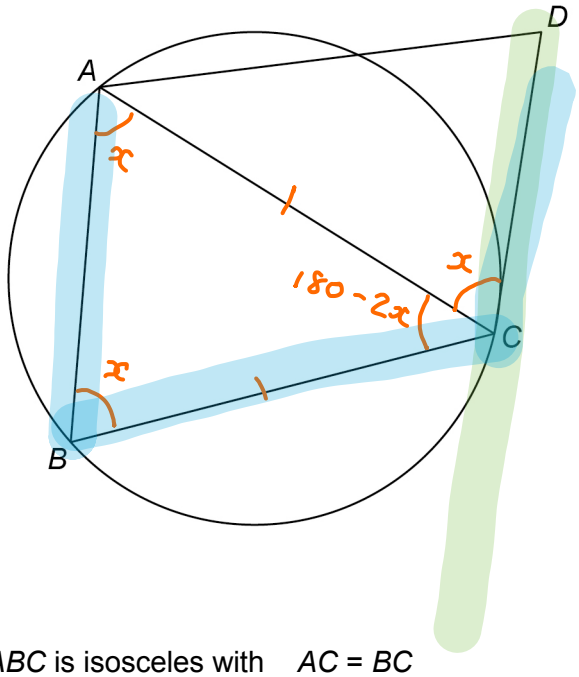
$\frac{1}{9}$ $\frac{1}{99}$ $\frac{1}{10}$ $\frac{11}{100}$

7

**20** *A*, *B* and *C* are points on a circle.

*CD* is a tangent.

Not drawn accurately




**20 (a)** Assume that triangle *ABC* is isosceles with $AC = BC$

Prove that *AB* is parallel to *DC*.

**[4 marks]**

This is the statement you need to end with

As ABC is isosceles $A\hat{B}C = B\hat{A}C = x$ (1)

so $A\hat{C}B = 180 - 2x$ (1)

As DC is a tangent $D\hat{C}A = A\hat{B}C = x$ due to alternate segment theorem. (1)

$D\hat{C}A + A\hat{B}C = 180 - 2x + x + x = 180$ (1)

∴ AB = DC must be parallel as interior/complementary angles add up to 180°

Do not write outside the box

**20 (b)** In fact, triangle *ABC* is equilateral. all angles 60°

Tick the **two** boxes for the statements that **must** be correct.

**[1 mark]**

- [x] *AB* is parallel to *DC* (1)
- [x] *AC* bisects angle *BCD*
- [ ] *AC* bisects angle *BAD*

**21** Solve the simultaneous equations

$$2x + 3y = 5p$$
$$y = 2x + p$$

where $p$ is a constant.

Give your answers in terms of $p$ in their simplest form.

**[4 marks]**

Through substitution

$2x + 3y = 5p$

$2x + 3(2x + p) = 5p$ (1)

$2x + 6x + 3p = 5p$

$8x + 3p = 5p$

$8x = 2p$

$x = \frac{1}{4}p$

If $x = \frac{1}{4}p$ (1)

$y = 2(\frac{1}{4}p) + p$

$= \frac{1}{2}p + p$

$= 1\frac{1}{2}p$

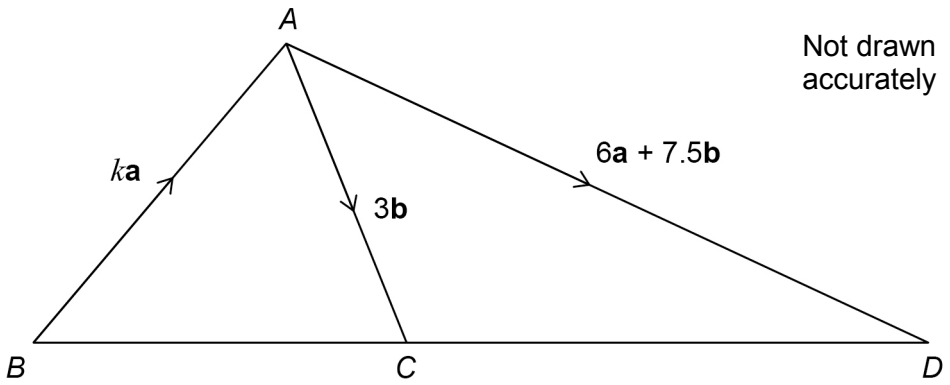
$x =$ 0.25p (1) $y =$ 1.5p (1)

9

Do not write outside the box

**22** *ABC* and *ACD* are triangles.

$k$ is a constant.




**22 (a)** Show that $\overrightarrow{CD} = 6\mathbf{a} + 4.5\mathbf{b}$

**[1 mark]**

$\vec{CD} = \vec{CA} + \vec{AD} = -3b + 6a + 7.5b = 6a + 4.5b$ (1)

**22 (b)** *BCD* is a straight line.

Work out the value of $k$.

You **must** show your working.

**[3 marks]**

If BCD is a straight line, $\vec{BC}$, $\vec{BD}$ and $\vec{CD}$ have common scalars

$\vec{CD} = 6a + 4.5b$

$\vec{BC} = k + 3b$ ×1.5 ×2.5

$\vec{BD} = (6+k)a + 7.5b$ (1)

$1.5k = 6$

$k = 4$

or (1)

$2.5k = 6 + k$

$1.5k = 6$

$k = 4$

Answer 4 (1)

Do not write outside the box

**23** Simplify $8^4 \div 32^{\frac{2}{5}}$

Give your answer in the form $2^m$ where $m$ is an integer.

**[3 marks]**

Change all to powers of 2 then simplify.

$8^4 = (2^3)^4 = 2^{12}$ (1)

$32^{2/5} = (2^5)^{2/5} = 2^2$ (1)

$2^{12} \div 2^2 = 2^{10}$

Answer $2^{10}$ (1)

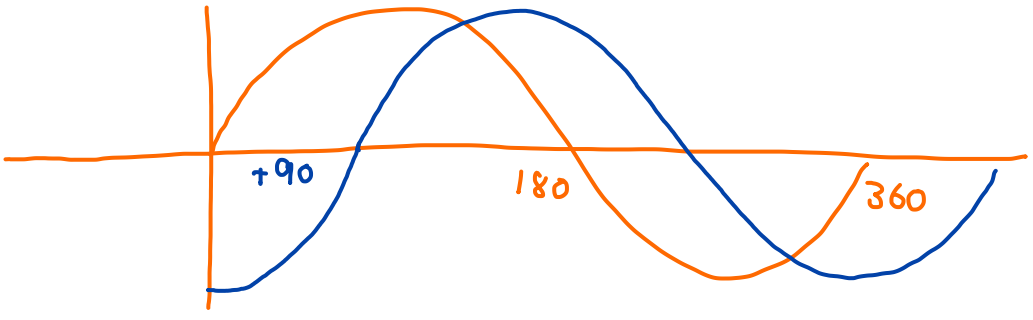
**24** $f(x) = \sin(x - 90°)$ ← Brackets lie

Circle the value of $f(0°)$

**[1 mark]**

1 0 $-\frac{1}{2}$ −1

Visualise the curve




**Turn over for the next question**

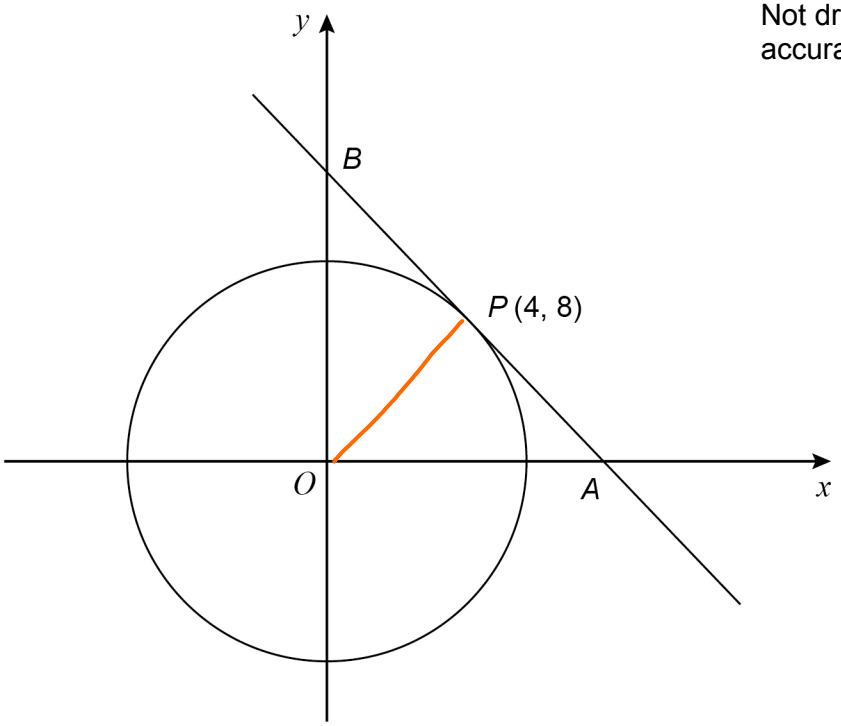
8

Do not write outside the box

**25** $P$ (4, 8) is a point on a circle, centre $O$.

The tangent at $P$ intersects the axes at points $A$ and $B$.

Not drawn accurately




**25 (a)** Show that the gradient of the tangent is $-\frac{1}{2}$

**[2 marks]**

Reach this conclusion.

Gradient of the radius → $\frac{8}{4} = 2$ (1)

Gradient of the tangent $= -\frac{1}{2}$

as $2 \times -\frac{1}{2} = -1$ (1)

Do not write outside the box

**25 (b)** Work out the length *AB*.

Give your answer in the form $a\sqrt{5}$ where $a$ is an integer.

You **must** show your working.

**[4 marks]**

Find point A and B by finding the equation of the tangent.

tangent → P(4, 8) $m = -\frac{1}{2}$ (1)

so $8 = 4(-\frac{1}{2}) + c$

$8 = -2 + c$ so $c = 10$

∴ B(0, 10)

If $y = 0$ $0 = -\frac{1}{2}x + 10$ A(20, 0)

$x = 20$ (1)

Use pythagoras $\sqrt{10^2 + 20^2} = \sqrt{500}$ (1)

$= \sqrt{100}\sqrt{5}$

Answer $10\sqrt{5}$ (1) units

**Turn over for the next question**

6

Do not write outside the box

**26** The turning point of the graph $y = (x + a)^2 + b$ has $x$-coordinate $-2$

(3, 1) is another point on the graph.

Work out the $y$-coordinate of the turning point.

**[3 marks]**

At (3, 1) $1 = (3 + a)^2 + b$

$1 = 9 + 6a + a^2 + b$

$a = 2$ $1 = 9 + 12 + 4 + b$

Re-write the equation with $a = 2$ (1)

$y = (x + 2)^2 + b$

At (3, 1) (1) $b = -24$

$1 = (3+2)^2 + b$ $\therefore y = -24$ at turning point.

$1 = 25 + b$

Answer $y = -24$ (1)

Do not write outside the box

**27** Angle $x$ is acute.

$\cos x = \sin 60° \times \tan 30°$

Work out the size of angle $x$.

You **must** show your working.

**[3 marks]**

Check the calculator gives you an acute angle

Recall trig ratios

$\sin 60° = \sqrt{3}/2$ (1)

$\tan 30° = \sqrt{3}/3$

$\cos x = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{3} = \frac{3}{6} = \frac{1}{2}$

$\cos x = \frac{1}{2}$ (1) → work backwards with trig ratios to find the angle

$x = 60°$

Answer 60 (1) degrees

**END OF QUESTIONS**

6

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